Midterm I Guidelines and Practice Exam
Math 110, Spring 2013, UC Berkeley

No cheat sheets, books, or computing devices will be allowed.

1 Time and Place

The first midterm will take place Wednesday, February 20th, in Stanley 105 during the lecture hour (2–3 p.m.).

2 Guidelines

This midterm will consist of three parts that will be equally weighted. You'll be asked to do the following.

1. Re-prove a theorem (or a part of a theorem) from the list of theorems below.
2. Redo an exercise from the homework.
3. Do an exercise similar to those in chapter 1 or chapter 2.

For all three parts, you'll be asked to a) write down some of the definitions involved, b) label each step you take with the axiom that allows you to take this step, c) indicate which proof strategy you are using (contraposition, contradiction, etc.). The grading for these requirements will be negative; that is, we will remove points for failing to write the involved definitions or to correctly justify a step taken.

Below are some tips on how to best prepare for the test. Following these tips will also give you a very strong foundation for the rest of the classwork to come.

1. Write out the definitions covered until February 15th (included).
2. Memorize the definitions.
3. Re-prove all theorems from the list below with the same level of detail as in the lecture (i.e., with each step carefully labeled with the correct axiom).

4. Memorize the theorem statements as well as their proofs.
5. Redo all exercises from assignments 1, 2, and 3.
6. Do the exercises from chapters 1 and 2 that were not covered in the homework.

3 List of Theorems

Proposition 1.2, Proposition 1.3, Proposition 1.4, Proposition 1.5, Proposition 1.6, Proposition 1.8, Proposition 1.9, Theorem 2.6, Theorem 2.10, Theorem 2.16, Theorem 2.14, Proposition 3.1, Proposition 3.2, Proposition 3.3, Theorem 3.4, Proposition 3.17. This list, which has grown until February 15th (included) is now complete. GOOD LUCK!!!!

4 Practice Exam

Question 1 Write out the definitions of subspace and direct sum. Suppose that U and W are subspaces of a vector space V such that $V = U + W$ and $U \cap W = \{0\}$. Prove that $V = U \oplus W$.

Question 2 Prove that if $(v_1, \ldots, v_n)$ spans V, then so does the list $(v_1 - v_2, v_2 - v_3, \ldots, v_{n-1} - v_n, v_n)$, which is obtained by subtracting from each vector (except the last one) the following vector.

Question 3 Write out the definition of linear independence. Suppose that $p_0, p_1, \ldots, p_m$ are polynomials in $\mathcal{P}_m(\mathbb{F})$ such that $p_j(2) = 0$ for each j. Prove that $(p_{0,}p_1, \ldots, p_m)$ is not linearly independent in $\mathcal{P}_m(\mathbb{F})$.